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OPTIMIZATION OF WELDED STRUCTURES FOR COST

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Abstract. Weight or mass minimization is a usual way of finding optima of structures. Due to the high cost of welding, weight and cost optima can be quite different. We have worked out a cost calculation for welded steel structures, considering not only the material, but preparation, cutting, edge grinding, welding, cleaning, painting costs as well. We have used the backtrack combinatorial discrete optimization method for many years. We made weight minimization. For cost minimization we had to modify the algorithm in order to be able to handle cost function. This paper describes this modification. We show two examples for cost minimization: the first example is cost optimization of welded box beams with longitudinal stiffeners. First we show the economics of using stiffeners in the example. The second example is the effect of post-welding treatments on the optimum fatigue design of welded I-beams. We show how cost saving can be achieved by introducing additional treatment.

Keywords: Structural optimization, cost calculation, welded structures, backtrack method, post-welding treatment

1. Introduction

Welding is a relatively expensive production technology, therefore it is important to decrease the cost of welded structures. Designers select a suitable structural version by comparison of several candidate structural solutions, but only optimized versions can be realistically compared to each other. Thus, the structural optimization is a basis not only for achieving savings in weight and cost, but also to help designers to select the most suitable structural version.

Box beams are widely used in load-carrying structures because of their large bending and torsional stiffness. The minimum cross-sectional-area design of a simple welded box beam shows that, in order to decrease this area, the web plate slenderness should be increased. This can be achieved using longitudinal stiffeners placed in 1/5 distance of web height. Although these stiffeners increase the weight and cost, significant savings can be achieved using them. To show these savings the minimum weight and cost design should be worked out.

Fatigue fracture is one of the most dangerous phenomena for welded structures. Welding causes residual stresses and sharp stress concentrations around the weld, which are responsible for significant decrease of fatigue strength. Butt welds with partial penetration, toes and roots of fillet welds are points where fatigue cracks initiate and propagate.

2. The cost function

We have developed a cost function for welded structures using the COSTCOMP database [1,2]. This cost function contains nonlinear expressions for unknowns. The cross-sectional area minimization can be solved analytically, but for the minimum cost design, a computerized mathematical constrained function minimization method should be used, since the cost function is nonlinear.

In the cost function the material and fabrication costs are included

$$K = K_m + K_f = k_m \rho V + k_f \sum_i T_i \,, \tag{2.1}$$

where ρ is the material density, V is the volume of the structure, k_m and k_f are the corresponding cost factors, T_i are the fabrication times. (2.1) can be written in the form of

$$\frac{K}{k_m} = \rho V + \frac{k_f}{k_m} \sum_i T_i \,. \tag{2.2}$$

We use the following cost factors: $k_m = 0.5 \div 1$ \$/kg, $k_{f\,\max} = 60$ \$/h $= 1$ \$/min, thus the ratio of k_m/k_f can be varied in a wide range of $0 \div 2$ kg/min. $k_m/k_f = 0$ means that K/k_m is a weight (mass) function, $k_m/k_f = 2$ kg/min can be used for developed countries.

The fabrication times can be calculated as follows:

$$\sum_i T_i = T_1 + T_2 + T_3 + T_4 \,. \tag{2.3}$$

Time for preparation, assembly and tacking is

$$T_1 = C_1 \Theta_d \sqrt{\kappa \rho V} \,, \tag{2.4}$$

where $C_1 = 1$ min/kg$^{0.5}$, Θ_d is a difficulty factor expressing the complexity of a structure (planar or spatial, consisting of plates or tubes etc.), κ is the number of elements to be assembled.

Time for welding is

$$T_2 = \sum C_{2i} a_{wi}^n L_{wi} \,, \tag{2.5}$$

where $C_{2i} a_{wi}^n$ is given for different welding technologies and weld shapes according to COSTCOMP software [1] and [2], a_w is the weld size, L_w is the weld length.

The additional time for electrode changing, deslagging and chipping can be calculated as

$$T_3 = 0.3 T_2 \,. \tag{2.6}$$

The final form of the cost function contains T_1, T_2, T_3 and T_4 for post-welding treatment, or some other times for cutting, edge grinding, surface preparation, flattening, painting, etc.

It can be seen that this function contains nonlinear members of the unknown structural dimensions, therefore an advanced backtrack method should be used for the minimization.

3. The advanced backtrack method

The backtrack discrete programming method is suitable for problems with few unknowns, but till now, we have used it only for linear objective functions.

For nonlinear objective functions we have worked out a new advanced version. The backtrack method is a combinatorial programming technique, which solves nonlinear constrained function minimisation problems by a systematic search procedure. The advantage of the technique is that it uses only discrete variables, so the solution is usable. The general description of backtrack can be found in the works by Walker [3] and Golomb & Baumert [4]. Farkas & Jármai [5] applied this method to welded I-girder design.

The general formulation of a single-criterion nonlinear programming problem is the following:

minimize

$$f(x), \quad x_1, x_2, ..., x_N \tag{3.1}$$

subject to

$$g_j(x) \leq 0, \quad j = 1, 2, ..., P \tag{3.2}$$

$$h_i(x) = 0, \quad i = P+1, ..., P+M\,. \tag{3.3}$$

Here $f(x)$ is a multivariable nonlinear function, $g_j(x)$ and $h_i(x)$ are nonlinear inequality and equality constraints. The equality constraints should be transformed to inequality ones for the program to handle them:

$$h_i(x) < |\,\epsilon\,|\,, \quad i = P+1, ..., P+M \tag{3.4}$$

ϵ is a given small number.

The algorithm is suitable for finding optimum for problems with monotonically increasing or decreasing objective functions. Thus, the optimum solution can be found by increasing or decreasing the variables. Originally the procedure can find the minimum of the problem. If we are looking for maximum, we should introduce $-f(x)$. The search is time-consuming, because the procedure makes a detailed search.

To find the optimum for a single variable, many single variable search techniques are available. An efficient and suitable search method is the interval halving procedure. We assume that the objective function is monotonously decreasing if the variables are decreasing. At the line search, when only one variable is changing, the aim is to find the minimum feasible value of the variable, starting from the maximum value. All values of variables are calculated by the halving procedure, except the last variable, which is calculated from the objective function.

The starting point, i.e. the maximum value, should satisfy the constraints. When the investigation shows that the minimum value satisfies the constraints, then the solution is found. If not, the region is divided into two subregions with the middle value. If the constraints are satisfied with the middle value, then the upper region is feasible, all points there satisfy the constraints. In this case we should investigate the lower region, to find the border between the feasible and unfeasible regions.

In the backtrack method the variables are in a vector form $x = \{x_i\}^T$, $(i = 1, ..., n)$ for which the objective function $f(x)$ will be a minimum and which will also satisfy the design constraints $g(x) \geq 0$, $(j = 1, ..., P)$. For the variables, series of discrete values are given in an increasing order. In special cases the series may be determined by $x_{k,\min}, x_{k,\max}$ and by the constant steps Δx between them.

First a partial search is carried out for each variable and if all variations have been investigated, a backtrack is made and a new partial search is performed on the previous variable. If this variable is the first one: no variations have to be investigated (a number of backtracks have been made), then the process stops. The main phases of the calculation are as follows.

1. With a set of constant values of $x_{i,t}(i = 2, ..., n)$, the minimum $x_{i,m}$ value satisfying the design constraints is searched for. The interval halving method can be employed. This method can be employed if the constraints and the objective function are monotonous from the sense of variables.
2. As in the case of the first phase, the halving process is now used with constant values, and the minimum value, satisfying the design constraints is then determined.
3. The least value is calculated from the equation relating to the objective function $f(x)$ where f is the value of the cost function calculated by inserting the maximum x-values.

Regarding the value, three cases may occur as follows.

(3a) We decrease the variables step by step till the constraints are satisfied or till the minimum values are reached. If all variations of the x_n value have been investigated, then the program jumps to the previous variable x_{n-1} and decreases it step by step till x satisfies the constraints or till minimum values are reached.

(3b) If $x_{n,m} < x_{n,1}$, we backtrack to x_{n-1} .

(3c) If $x_{n,m}$ does not satisfy the constraints, we backtrack to $x_{n-1,m}$. If the constraints are satisfied, we continue the calculation according to (3a).

The number of all possible variations is $\prod_{i=1}^{n} t_i$ where t_i is the number of discrete sizes for one variable. However, the method investigates only a relatively small number of these. Since the efficiency of the method depends on many factors (number of unknowns, series of discrete values, position of the optimum values in the series, complexity of the cost function and/or that of the design constraints), it is difficult to predict the run time. The main disadvantage of the method is that the run time increases exponentially if we increase the number of unknowns.

The original version of backtrack was modified by rebuilding the algorithm so that it is independent from the number of variables, since in the original algorithm all variable values are calculated by the halving procedure, except the last one. Another development is that the Van Wijngaarden-Dekker-Brent method (Brent [6]) was built into the algorithm to calculate the last variable value from the cost function. In case of mass minimization this calculation is relatively easy, because of the linearity,

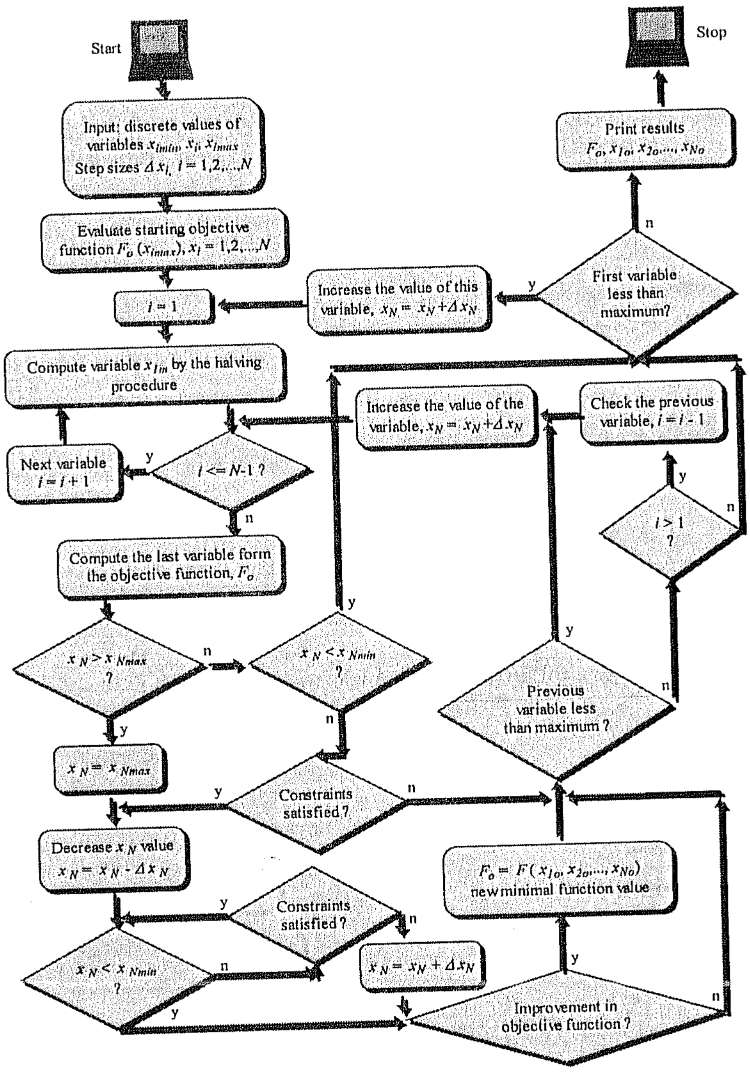

Figure 1. The flowchart of the backtract method

but introducing a nonlinear cost function the analytical solution in most cases is impossible. This method combines root bracketing, bisection and inverse quadratic interpolation to converge from the neighbourhood of a zero crossing. While the false position and secant methods assume approximately linear behaviour between two prior root estimates, inverse quadratic interpolation uses three prior points to fit an inverse quadratic function. This method combines the sureness of bisection with the speed of a high-order method when appropriate. This calculation is built into the computer code with subroutines. The flowchart of the backtrack method is shown in Figure 1.

4. Minimum cross-sectional-area for box beams with stiffeners

In this section we deal with the design of minimum cross-sectional-area for bending of box beams with longitudinal stiffeners. As can be seen from Figure 2 the cross-sectional area to be minimized is

$$A = 2ht_w + 2bt_f + 2t_s(b_1 + b_2)\,, \tag{4.1}$$

The stress constraint can be expressed as

$$\sigma_{\max} = \frac{M_{\max}}{W_x} \le f_y\,, \tag{4.2}$$

where $M_{\max}$ is the maximum bending moment, W_x is the section modulus, f_y is the yield stress. In the moment of inertia the effect of stiffeners is neglected, thus

$$I_x \simeq \frac{h^3 t_w}{6} + 2bt_f \left(\frac{h}{2}\right)^2 . \tag{4.3}$$

The stress constraint (4.2) can be written in the following form

$$W_x \simeq \frac{I_x}{\frac{h}{2}} = \frac{h^2 t_w}{3} + bt_f h \ge W_o = \frac{M_{\max}}{f_y}\,. \tag{4.4}$$

From (4.1) we obtain

$$bt_f = \frac{A}{2} - ht_w - A_S\,. \tag{4.5}$$

Substituting (4.5) into (4.4) we get

$$W_x = \frac{Ah}{2} - \frac{2h^2 t_w}{3} - A_S h \ge W_0\,. \tag{4.6}$$

In addition it follows from (4.6) that

$$A \ge \frac{2W_0}{h} + \frac{4h^2 t_w}{3} + 2A_S\,. \tag{4.7}$$

According to Eurocode 3 [7] the local buckling constraint for the compression flange can be expressed as

$$\frac{b}{t_f} \le \frac{1}{\delta} = 42\varepsilon \qquad \text{and} \qquad \varepsilon = \sqrt{\frac{235}{f_y}}\,. \tag{4.8}$$

The local buckling constraint for the upper part of the webs is

$$\frac{0.2h}{t_w} \le \frac{42\varepsilon}{0.67 + 0.33\psi}\,. \tag{4.9}$$

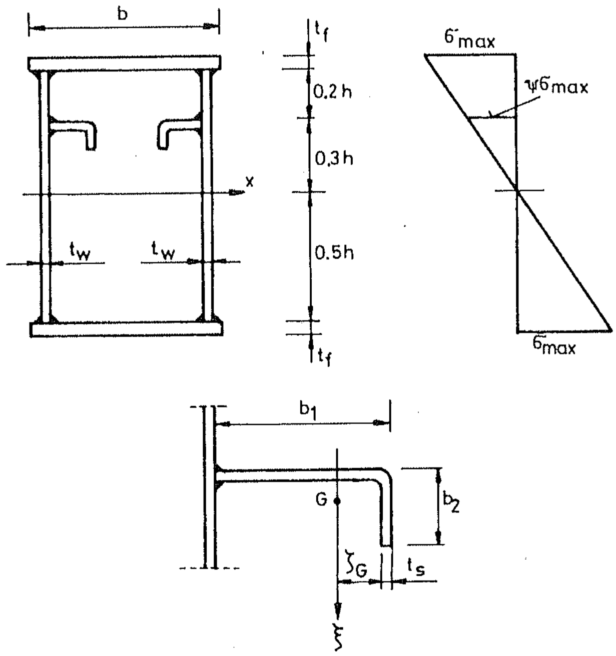

Figure 2. Stiffened box beam and the detail of a stiffener

If $\psi = 0.6$ we get

$$h/t_w \leq 242\varepsilon = 1/\beta\,. \tag{4.10}$$

For the lower part of the webs $\psi = -5/3 = -1.667$ and

$$0.8h/t_w \leq 62\varepsilon(1-\psi)\sqrt{-\psi} \tag{4.11}$$

or

$$h/t_w \leq 267\varepsilon \tag{4.12}$$

thus, the limiting slenderness defined by (4.10) is governing.

The local buckling constraints for the cold-formed stiffeners according to DASt Richtlinie 016 [8] are

$$b_1/t_s \leq 1.33\sqrt{E/f_y} \tag{4.13}$$

and

$$b_2/t_s \leq 0.43\sqrt{E/f_y} \tag{4.14}$$

or in other form

$$b_1/t_s \leq 30\varepsilon = \delta_1 \qquad \text{and} \qquad b_2/t_s \leq 12.5\varepsilon = \delta_2 \,. \tag{4.15}$$

According to API design rules [9] the overall buckling constraints for compressed stiffeners are of the form

$$I_\xi \geq 4at_w^3 \qquad \text{and} \qquad C_s = \frac{1}{12}\left(\delta_1^3 + \frac{3\delta_1^2\delta_2^2}{\delta_1+\delta_2}\right) \,, \tag{4.16}$$

where a is the distance of diaphragms. With

$$b_1 = \delta_1 t_S \qquad \text{and} \qquad b_2 = \delta_2 t_S \tag{4.17}$$

the centre of gravity for a stiffener (Figure 2) is

$$\xi_G = \frac{t_S \delta_1^2}{2(\delta_1+\delta_2)} \tag{4.18}$$

and the moment of inertia is given by

$$I_\xi = C_S t_S^4 \,. \tag{4.19}$$

Expressing a in terms of h

$$a = C_a h \tag{4.20}$$

and using (4.19), from (4.16) one obtains (4.21)

$$t_S = 4\sqrt{\frac{4C_a\beta^4}{C_S}} \,. \tag{4.21}$$

Consequently relation (4.7) can be written as

$$A = 2W_0/h + 4\beta h^2/3 + 2h^2 C_A \,, \tag{4.22}$$

$$C_A = (\delta_1+\delta_2)\sqrt{4C_a\beta^3/C_S} \,. \tag{4.23}$$

The condition $dA/dh = 0$ gives the optimum beam height

$$h_{opt} = \sqrt[3]{\frac{W_0}{4\beta/3 + 2C_A}} \,. \tag{4.24}$$

Without longitudinal stiffeners we obtain

$$h_{opt} = \sqrt[3]{\frac{3W_0}{4\beta_0}} \tag{4.25}$$

where

$$\frac{1}{\beta_0} = 124\varepsilon \tag{4.26}$$

as was already shown in the book [10] by Farkas. Expressing h_{opt} from (4.24), or from (4.25) and substituting it into (4.22) we get

$$A_{\min} = \sqrt[3]{18W_0^2(2\beta + 3C_A)}\,. \tag{4.27}$$

If there are no stiffeners

$$A_{0.\,\min} = \sqrt[3]{36\beta_0 W_0^2}\,. \tag{4.28}$$

Comparison of beams with and without stiffeners: taking $f_y = 235$ MPa, we calculate with the following values: $1/\beta = 242$, $1/\beta_o = 124$, $\delta_1 = 30$, $\delta_2 = 12.5$, $C_s = 3077.2$, in (4.22) taking $C_a = 1.5$ we get $C_A = 1/2006$.

With (4.24) and (4.25) we obtain

$$h_{opt} = 5.36\sqrt[3]{W_0} \tag{4.29}$$

and

$$h_{0.opt} = 4.53\sqrt[3]{W_0}\,, \tag{4.30}$$

respectively, i.e., a beam with longitudinal stiffeners has 15% higher webs than without stiffeners.

According to (4.27)

$$A_{\min} = 0.5601\sqrt[3]{W_0} \tag{4.31}$$

and with (4.28)

$$A_{0.\,\min} = 0.6622\sqrt[3]{W_0}\,, \tag{4.32}$$

i.e., a beam with stiffeners has 18% smaller weight than without stiffeners.

5. Minimum cost design of longitudinally stiffened box beams

A simply supported beam of span length $L = 20$ m is subjected to uniformly distributed factored normal load of intensity $p = 73.5$ N/mm. It is assumed that the beam is constructed with 11 transverse diaphragms of uniform distance $a = 2$ m to stabilize the stiffeners against flexural buckling and to avoid distortions of the rectangular box shape. The longitudinal stiffeners are interrupted and welded to diaphragms.

The volume of the structure is

$$V = AL + 11bht_D/4 + 2A_SL \tag{5.1}$$

where

$$A = 2ht_W + 2bt_f \ . \tag{5.2}$$

The thickness of diaphragms is $t_D = 0.7t_W$, but rounded to 4, 5 or 6 mm.

The cross-sectional area of a stiffener is

$$A_S = (b_1 + b_2)t_S \tag{5.3}$$

in which b_1 and b_2 can be calculated according to the equations $(4.17)_{1,2}$.

The number of structural elements to be assembled is $\kappa = 4 + 11 + 20 = 35$. The difficulty factor is taken to be $\Theta_d = 3$.

The following welding times are considered.

For longitudinal fillet welds of size $a_w = 0.5t_w$, SAW (submerged arc welding), $a_w = 0 \div 15$ mm

$$T_{21} = 0.2349L(0.5t_w)2\text{x}10^{-3} \quad (L \text{ in mm}) \ . \tag{5.4}$$

For transverse fillet welds of constant size $a_w = 4$ mm connecting the diaphragms to the box section, SMAW (shielded metal arc welding)

$$T_{22} = 0.788911(b + 2h)42\text{x}10^{-3} \quad (b \text{ and } h \text{ in mm}) \ . \tag{5.5}$$

For the two longitudinal fillet welds of size $a_w = 4$ mm connecting the stiffeners to the webs GMAW-C (gas metal arc welding with CO_2)

$$T_{23} = 0.3394\text{x}2L\text{x}42x10^{-3} \quad (L \text{ in mm}) \tag{5.6}$$

For transverse fillet welds of size $a_w = 4$ mm connecting the stiffeners to the diaphragms, SMAW

$$T_{24} = 0.7889(42.5t_S\varepsilon)4^2x20x10^{-3} \quad (t_S \text{ in mm}) \ . \tag{5.7}$$

6. Design constraints

Stress constraint due to bending

$$\frac{M}{W_x} = \frac{pL^2}{8W_x} \le f_y \tag{6.1}$$

where the first moment and the moment of inertia for the cross secrion are

$$W_x = \frac{2I_x}{h + t_f} \quad \text{and} \quad I_x = \frac{h^3 t_w}{6} + \frac{bt_f(h + t_f)^2}{2} \ . \tag{6.2}$$

Note that the moment of inertia of stiffeners is neglected.

The local buckling constraints for box beam webs and flange are given by equations (4.17). The constraint on flexural buckling of stiffener parts with buckling length of

$a = 2000$ mm is given by equation (4.16). Using equations (4.17) and (4.19) the moment of inertia of a stiffener can be calculated as

$$I_\xi = 3077\varepsilon^3 t_S^4 \tag{6.3}$$

The unknowns in the optimization are as follows: h, b, t_w, t_f, t_S.

7. Optimization and results

The optimization of the box beam is performed by the Hillclimb and backtrack methods. In the case of longitudinally stiffened box beam the cost function (2.2) should be minimized considering the constraints (4.2, 4.8, 4.10, 6.1). In the case of box beams without stiffeners the following modifications should be used: $t_S = 0$ and $T_{23} = T_{24} = 0$. The results are given in Tables 1 and 2.

Table 1. Optimum dimensions in mm of the box beam without longitudinal stiffeners obtained by the Hillclimb method

k_f/k_m	h	t_W	b	t_f	K/k_m (kg)
0	1100	9	540	20	6580
1	900	8	740	20	9249
2	910	8	730	20	11474

It can be seen that the results obtained by the Hillclimb and backtrack methods are nearly the same, so the new version of backtrack is suitable for nonlinear objective functions. The comparison of results obtained for unstiffened and stiffened box beams shows cost savings of 18-21%, so the application of longitudinal stiffeners is economical.

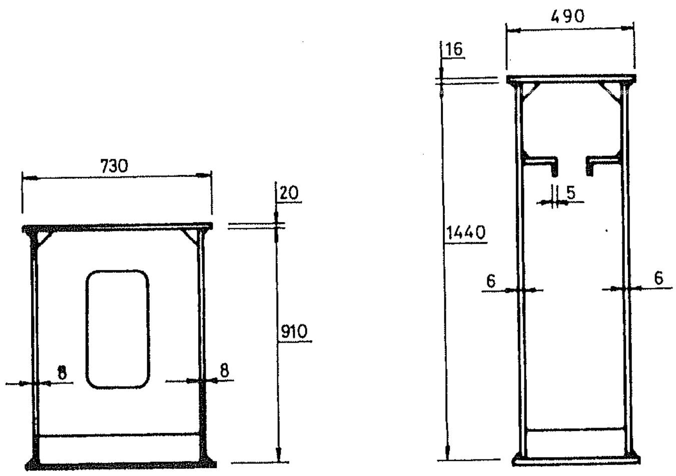

Figure 3. Optimum sizes of box beams without and with longitudinal stiffeners, for $k_f/k_m = 2$

Figure 3 shows the optimized box beams without and with longitudinal stiffeners. It can be seen that the beam with stiffeners is higher and narrower.

Table 2. Optimum dimensions in mm of the box beam with longitudinal stiffeners obtained by the Hillclimb and backtrack method

Method	k_f/k_m	h	t_W	b	t_f	t_S	K/k_m (kg)
	0	1450	6	440	18	5	5610
Hillclimb	1	1450	6	490	16	5	7588
	2	1440	6	500	16	5	9619
	0	1450	6	490	16	5	5591
Backtrack	1	1440	6	470	17	5	7608
	2	1440	6	490	16	5	9585

8. Post-welding treatments and optimum fatigue design of welded I-beams

In order to eliminate or decrease the danger of fatigue fracture several methods have been investigated. Post-welding treatments (PWT-s) such as toe grinding, TIG-dressing, hammer peening and ultrasonic impact treatment (UIT) are the most efficient methods. These methods have been tested and a lot of experimental results show their effectiveness and reliability.

For designers it is important to know the measure of saving in structural weight and cost, which can be achieved by using these treatments. Optimum design is suitable for this task, since the additional cost of PWT can be included in the cost function and the improved fatigue stress range can be considered in the fatigue strength constraint. Thus, our aim is to illustrate this saving by means of a simple numerical example of a welded I-beam.

In this case the transverse fillet welds used for vertical stiffeners decrease the fatigue stress range, thus the effect of PWT can be illustrated minimizing the cost function, which contains also the additional cost of PWT and the increased fatigue stress range can be included in the fatigue stress constraint. Note that Farkas [11] has treated this problem in a recent article for a welded box beam using only a few experimental data given by Woodley [12].

Table 3. Some improvement data according to [13]

	Stress range (MPa) at 2×10^6	Improvement % at 2×10^6
As welded	86	–
UIT	190	121
TIG dressing	132	53
TIG+UIT	202	135

Haagensen et al [13] have summarized the results of investigations relating to the measure of improvement in a table, from which we cite some basic data in Table 3. Note that the data are obtained for high strength steel of yield stress 780 MPa.

A wide overview of results is given by Braid et al [14]. This article gives a hammer-peening speed of 25 mm/s and uses 6 passes, i.e., 6x1000/(25x60) = 4 min/m.

Maddox et al [15] have given the improvement citing the UK standard fatigue classes stating that the fatigue limit for weld toe burr grinding or hammer peening equals the UK class C at $2\text{x}10^6$ cycles. According to BS 5400 Part 10 (1980) [16] for transverse fillet welds in as welded state the fatigue limit is given by Class F of 40 MPa at 10^7 cycles, and for Class C of 78 MPa. Calculation for $2\text{x}10^6$ cycles gives 68 and 123 MPa, respectively, thus, the improvement is 123/78 = 1.8 (80%).

Lobanov and Garf [17] have treated the effect of UIT in connections of tubular structures.

According to Gregor [18] TIG-dressing results in 40% improvement. Woodley [12] gives also 40% improvement for toe burr grinding and the necessary time for grinding 60 min/m.

According to Janosch et al [19] the ultrasonic peening of fillet welded T-joints results in a fatigue stress range at $2\text{x}10^6$ cycles of 290 MPa, which is 70-80% improvement compared to the as-welded value of 168 MPa. For a treatment of 3 passes 15 min/m specific time has been necessary.

Huther et al [20] worked out a summary of improvement methods and results using data of 51 references. For fillet welded T- or cruciform joints the following final design fatigue stress ranges at $2x10^6$ cycles can be used: for TIG dressing 124 MPa (70% improvement as compared to EC3 data); for hammer peening 209 MPa (190% improvement). These data are valid for steels of yield stress less than 400 MPa.

For our purpose those publications are suitable in which data are given not only for the measure of improvement (α), but also for the time required for treatment (T_0). These data are summarized in Table 4.

Table 4. Measure of improvement and specific treatment time for various treatments according to the published data

Method	Reference	T_0 (min/m)	Improvement %	α	Remark
Grinding	[12]	60	40	1.4	
TIG dressing	[21]	18	40	1.4	70-100%
Hammer peening	[14]	4	100	2.0	175-190%
UIT	[19]	15	70	1.7	

It should be mentioned that we want to calculate with the minimum value of improvement. A value larger than 100% cannot be realized in our numerical example.

9. Minimum cost design of a welded I-beam considering the improved fatigue stress range and the additional PWT cost

In the investigated numerical example transverse vertical stiffeners are welded to a welded I-beam with double fillet welds. PWT is used only in the middle of the span,

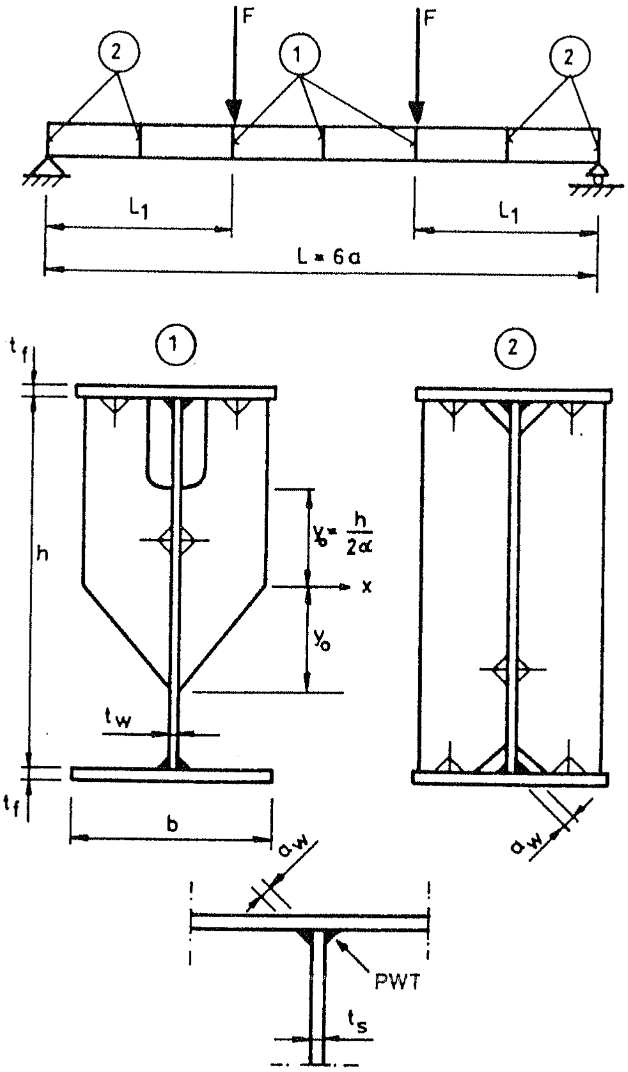

Figure 4. Welded I-beam with vertical stiffeners. Double fillet welds with (1) and without (2) PWT

since near the supports the bending stresses are small. The tension part of stiffeners in the middle of span is not welded to the lower flange and to the lower part of the web. Thus, PWT is needed only for welds connecting the stiffeners to the upper flange (Figure 4). For this reason two types of stiffeners are used as it can be seen in Figure 4.

The beam is loaded by a pair of forces fluctuating in the range of $0 \div F_{max}$, so the bending stress range is calculated from F_{max} .

Time for PWT is

$$T_4 = T_0 L_t \tag{9.1}$$

where T_0 is the specific time (min/mm) and L_t is the treated weld length (mm).

The final form of the cost function is as described in [22]

$$\frac{K}{k_m} = \rho V + \frac{k_f}{k_m}\left(\Theta_d\sqrt{\kappa\rho V} + 1.3\sum C_{2i}a_{wi}^n L_{wi} + T_0 L_t\right) . \tag{9.2}$$

10. Design constraints

The constraint on fatigue stress range can be formulated as

$$\frac{F_{\max}L_1}{W_x} \le \frac{\alpha\Delta\sigma_C}{\gamma_{Mf}} \tag{10.1}$$

where

$$W_x = \frac{I_x}{\frac{h}{2}+\frac{t_f}{2}}; I_x = \frac{h^3 t_w}{12} + 2bt_f\left(\frac{h}{2}+\frac{t_f}{2}\right)^2 . \tag{10.2}$$

According to Eurocode 3 (EC3) the fatigue stress range for as welded structure is $\Delta\sigma_C = 80$ MPa, the fatigue safety factor is $\gamma_{Mf} = 1.25$.

The quotient α expresses the measure of improvement

$$\alpha = \frac{\Delta\sigma_{Cimproved}}{\Delta\sigma_{Caswelded}} . \tag{10.3}$$

The constraint on local buckling of the web according to EC3 is

$$\frac{h}{t_w} \le 69\varepsilon; \qquad \varepsilon = \sqrt{\frac{235}{\alpha\Delta\sigma_C/\gamma_{Mf}}} \tag{10.4}$$

Note that we calculate in the denominator of ε with the maximum compressive stress instead of yield stress [23].

The constraint on local buckling of the compression flange is

$$b/t_f \le 28\varepsilon . \tag{10.5}$$

11. Numerical example

Data: $F_{\max} = 138$ kN, $L = 12$ m, $L_1 = 4$ m, $\Delta\sigma_C/\gamma_{Mf} = 80/1.25 = 64$ MPa, $\varepsilon = 1.916/\sqrt{\alpha}$; $\Theta_d = 3$. The number of stiffeners is 2x7 = 14, thus $\kappa = 3 + 14 = 17$.

The volume of the structure is

$$V = (ht_w + 2bt_f)L + 4bht_S + 1.5bht_S\left(1+\frac{1}{\alpha}\right) \tag{11.1}$$

in which $t_s = 6$ mm.

The second member expresses the volume of stiffeners without PWT, the third member gives the volume of stiffeners with PWT.

For longitudinal GMAW-C (gas metal arc welding with CO_2) fillet welds of size 4 mm we calculate with

$$C_2 a_w^n L_w = 0.3394\text{x}10^{-3}\text{x}42\text{x}4L = 260 \text{ mm}, \tag{11.2}$$

for transverse SMAW (shielded metal arc welding) fillet welds the following formula holds

$$C_2 a_w^n L_w = 0.7889\text{x}10^{-3}\text{x}4^2 \left[6\left(b + \frac{2h}{\alpha}\right) + 16(b+h)\right]. \tag{11.3}$$

For the constrained minimization of the nonlinear cost function the Rosenbrock Hill-climb mathematical programming method is used complemented with an additional search for optimum rounded discrete values of unknowns. The results of computation, i.e. the unknown dimensions h, t_w, b and t_f as well as the minimum costs for different values of k_f/k_m and α are given in Table 5.

Table 5. Optimum rounded dimensions in mm and K/k_m (kg) values for different k_f/k_m ratios for various PWT-s. $k_f/k_m = 0$ means the minimum weight design without effect of PWT

PWT	k_f/k_m (kg/min)	h	t_w	b	t_f	K/k_m (kg)
as	0	1300	10	320	14	2191
welded	1	1230	10	310	16	3802
	2	1230	10	310	16	5399
Grinding	1	940	9	340	15	3343
	2	890	8	300	19	4704
TIG	1	1000	9	330	14	3235
dressing	2	1110	10	310	12	4770
Hammer	1	820	9	310	13	2762
peening	2	820	9	310	13	3999
UIT	1	970	10	300	12	3021
	2	810	8	300	17	4202

12. Conclusions

In this paper the benefits of using optimization methods for welded structures are presented. The two examples shown illustrate the possibility of cost and mass reduction. In the first example comparisons of the optimized cross-sectional areas and costs of box beams without and with longitudinal stiffeners placed at a distance of 1/5 web height show that the use of stiffeners results in considerable savings. The minimum cross-sectional area design is solved analytically deriving closed formulae for the optimum beam dimensions. The cost function contains material and welding costs, considering also the welds necessary for transverse diaphragms. Since this cost function is nonlinear, a new version of backtrack discrete combinatorial programming method is developed and used. This version contains a subroutine for the computation of the roots of a nonlinear function (cost function) with one variable. Results obtained by Hillclimb and backtrack methods are nearly the same, so the new version of backtrack is suitable for nonlinear objective functions. The comparison of results obtained for unstiffened and stiffened box beams shows cost savings of 18-21%, so the application of longitudinal stiffeners is economical.

In the second example a welded I-beams with vertical stiffeners was optimized using double fillet welds with and without post-welding treatment. It can be seen, that using various treatment methods one can achieve cost saving even if these treatments are expensive. The example shows that elimination or decreasing the danger of fatigue fracture in welded structures can be connected to cost saving. The following cost savings can be achieved: grinding 14-15 %, TIG dressing 13-17 %, hammer peening 35-38 %, UIT 26-28 %. Thus, the cost savings are significant, and the most efficient method is hammer peening. It can be also seen that PWT methods affect the optimum dimensions.

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